DATA ANALYSIS AND DETECTION METHODS FOR ON-LINE HEALTH MONITORING OF BRIDGE STRUCTURES

S.-L. James Hu, Timothy S. Hillier and Peter Stepanishen

June 2002

URITC 536112

PREPARED FOR
UNIVERSITY OF RHODE ISLAND
TRANSPORTATION CENTER

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### Title and Subtitle
Data Analysis and Detection Methods for On-Line Health Monitoring of Bridge Structures

### Authors(s)
Sau-Lon James Hu, Timothy S. Hillier & Peter Stepanishen

### Performing Organization Name and Address
University of Rhode Island
Dept. of Ocean Engineering
South Ferry Road
Narragansett, RI 02882-1197

### Sponsoring Agency Name and Address
University of Rhode Island Transportation Center
85 Briar Lane
Kingston, RI 02881

### Abstract
Developing an efficient structural health monitoring (SHM) technique is important for reducing potential hazards posed to the public by damaged civil structures. The ultimate goal of applying SHM is to real-time detect, localize, and quantify the accumulated structural damage. Traditionally, damage assessment techniques are primarily based on the observation of the changes of model parameters, such as modal frequencies and vibration shapes. However, several investigations have pointed out that this technique could be efficient only if very precise measurements are available or large level of damage exists. It is obvious that a more robust damage assessment algorithm must be developed. The objective of this research is to develop a new technique, named matched damage processing (MDP) technique, to perform damage detection, localization, and severity estimation for bridge structures subjected to traffic loading. The MDP technique builds upon the concept of matched field processing already developed in the underwater acoustics community. In short, the MDP technique compares measured responses collected from the physical structure with multiple modeled results calculated based on various damage locations and severity. The comparison is displayed in a contour or image plot with damage location and severity as the independent coordinates. The concept, effectiveness and robustness of the MDP technique are numerically demonstrated via a simple example where a simply supported beam is used to model a bridge and the traffic loading is modeled as a constant weight moving with constant speed across the bridge. A localized reduction of structural stiffness is applied to represent the occurrence of damage to the bridge. The mode superposition technique in conjunction with the finite element method is employed to solve the numerical problem. According to the numerical results, it is found that the MDP technique is effective and robust to estimate location and severity of structural deterioration.
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Data Analysis and Detection Methods for On-Line Health Monitoring of Bridge Structures

S.-L. James Hu†, Timothy S. Hillier‡ and Peter Stepanishen§

Abstract

Developing an efficient structural health monitoring (SHM) technique is important for reducing potential hazards posed to the public by damaged civil structures. The ultimate goal of applying SHM is to real-time detect, localize, and quantify the accumulated structural damage. Traditionally, damage assessment techniques are primarily based on the observation of the changes of model parameters, such as modal frequencies and vibration shapes. However, several investigations have pointed out that this technique could be efficient only if very precise measurements are available or large level of damage exists. It is obvious that a more robust damage assessment algorithm must be developed. The objective of this research is to develop a new technique, named matched damage processing (MDP) technique, to perform damage detection, localization, and severity estimation for bridge structures subjected to traffic loading. The MDP technique builds upon the concept of matched field processing already developed in the underwater acoustics community. In short, the MDP technique compares measured responses collected from the physical structure with multiple modeled results calculated based on various damage locations and severity. The comparison is displayed in a contour or image plot with damage location and severity as the independent coordinates. The concept, effectiveness and robustness of the MDP technique are numerically demonstrated via a simple example where a simply supported beam is used to model a bridge and the traffic loading is modeled as a constant weight moving with constant speed across the bridge. A localized reduction of structural

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*This research is supported by URITC, Project Number: 5-36112
†Professor, Dept. of Ocean Engrg., Univ. of Rhode Island, Narragansett, RI 02882-1197, Tel: 401-874-6688, Fax: 401-874-6837, E-mail: hu@oce.uri.edu
‡Graduate student, Dept. of Ocean Engrg., Univ. of Rhode Island, Narragansett, RI 02882-1197
§Professor, Dept. of Ocean Engrg., Univ. of Rhode Island, Narragansett, RI 02882-1197
stiffness is applied to represent the occurrence of damage to the bridge. The mode superposition technique in conjunction with the finite element method is employed to solve the numerical problem. According to the numerical results, it is found that the MDP technique is effective and robust to estimate location and severity of structural deterioration.

**Introduction**

It is always desirable for an inspection method to assess, on a periodic or constant basis, the condition of a structure that might have suffered a variety of damages. Structural health monitoring (SHM) systems, which are any nondestructive damage evaluation (NDE) technology for damage detection, location and/or assessment using structural response data, intend to achieve this objective. Nondestructive damage evaluation techniques can be classified as local techniques that focus on specific structural components, and global techniques that assess the condition of the entire structure. Local NDE techniques include those involving acoustics, dye penetrant, emission spectroscopy, leak testing, optics, magnetic particles, magnetic perturbation, pressure/vacuum testing, pulse-echo, radiography, ultrasonics, X-ray, and visual inspection. Most of these techniques have been used successfully to detect location of certain elements, cracks or weld defects, corrosion/erosion, and so on; and many of these methods are routinely applied as part of regular or remedial maintenance programs. However, local health monitoring methods require that the vicinity of the damage is known *a priori* and that the portion of the structure being inspected is readily accessible [11]. The development of a global monitoring system would avoid such obstacles.

This study focuses on global health monitoring. Its ultimate goal is to real-time detect, localize, and quantify the damage as it occurs. Currently, there is much research on the subject of global damage detection techniques using changes in the dynamic properties of response of structures. Modal parameters, notably frequencies, mode shapes, and modal damping, are a dynamic function of the physical properties of the structure (mass, damping, stiffness and boundary conditions) [16]. Damage to the structure will alter these physical properties and therefore analyzing changes in these parameters is an inviting option. Such evaluation techniques have been used extensively in many disciplines. For instance, mechanical engineers routinely employ vibration methods to monitor the condition of rotating machinery. In aerospace engineering, global health monitoring methods are used to detect cracking in airframes as well as controlling motion in space structures. Due to improvements in instrumen-
tation and comprehension of dynamics of complex structures, health monitoring and
damage assessment of civil structures has become more practical in the systematic
inspection and evaluation of these structures.

In practice, highway bridges are often inspected on a biennial basis, largely with
the use of visual inspection techniques. However, damage could go undetected at
inspection or cracks in load-carrying member could grow to critical levels between
inspection intervals. As a result, Mazurek and DeWolf [20] present strong argu-
ments in favor of a continuous automated vibration monitoring system for highway
bridges, citing several unexpected collapses and near collapses. Obviously, a quantita-
tive mechanism of bridge damage detection is necessary. Over the past twenty years,
there have been repeated attempts at applying variations in the modal parameters of
bridges to the fields of damage detection and structural monitoring, and many papers
have been published on these methods. A comprehensive literature review has been
published by the Los Alamos National Laboratory [12]. However, recent investiga-
tions have pointed towards limitations of damage detection via modal parameters.
Frequency shifts offer somewhat low sensitivity to damage. Utilizing frequency shifts
would therefore require very precise measurements or large levels of damage. Farrar,
et al. [13] presented the results of a damage-detection experiment performed on the
I-40 bridge over the Rio Grande river in Albuquerque, NM. The authors found that
modal frequency is not a sensitive indicator of damage, as it took a large reduction
in the bridge bending stiffness to see any changes in the measured modal frequencies
or mode shapes. Furthermore, because modal frequencies are global parameters of
the structure that cannot provide spatial information about structural changes, one
could not rely on the observation of frequency changes to locate the damage location.
Observing higher order modes can provide information about local changes, but the
practical limitations could make them difficult to identify. It is obvious that a more
robust damage detection method must be developed.

In this study a new global technique — matched damage processing (MDP) — is
introduced to perform damage detection for bridge structures. The underlying prin-
ciple of the MDP is similar to that of the matched field processing (MFP) used by
the underwater acoustic community [27]. It basically compares the realistic structural
response measurements with their computed counterparts from various damaged or
undamaged numerical models. The output of the MDP is usually displayed as a con-
tour or image plot that provides visual indication for the damage location and severity.
The concept of MDP will be demonstrated via a simple numerical example where a
one-dimensional physical model (a simply supported beam) of a bridge is subjected to
“traffic” loading. For a homogeneous beam (undamaged model), closed form solution
for the dynamic response can be obtained analytically. However, to obtain the dynamic response for an inhomogeneous beam (damaged bridge), a numerical approach must be applied, in which the finite element method in conjunction with the mode superposition technique will be employed to solve the numerical problem. A localized reduction of structural stiffness is applied to represent the occurrence of damage to the bridge.

Mathematical Model and Solution Procedure

The practical objective of this research is to develop a damage detection algorithm for bridge structures excited by traffic loading. For simplicity, the physical model for the bridge structure is assumed to be a simply supported beam. Mathematically, this physical model is characterized by the Euler-Bernoulli beam equation:

$$\frac{m}{\bar{m}} \frac{\partial^2 v(x, t)}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left[ EI \frac{\partial^2 v(x, t)}{\partial x^2} \right] = q(x, t), \quad 0 \leq x \leq L, \quad 0 \leq t \leq \infty$$

(1)

constrained by the boundary conditions: $v(0, t) = v(L, t) = 0$, where $v(x, t) = \text{transverse displacement of the beam}$, $\bar{m} = \text{mass density per unit length}$, $EI = \text{beam rigidity}$, $q(x, t) = \text{externally applied transverse pressure loading}$, $L = \text{length of the beam}$, and $t$ and $x$ indicate time and the horizontal location along the beam.

In this study, the traffic loading is assumed to be caused by a vehicle of constant mass, traveling across the beam with a constant velocity. Denoting $W$ as the constant weight, and $c$ the constant speed of the vehicle, one can express the traffic loading as:

$$q(x, t) = W\delta(x - ct), \quad 0 \leq x \leq L, \quad 0 \leq t \leq \frac{L}{c}$$

(2)

where $\delta$ stands for the Dirac delta function.

Analytical Solution Procedure

For the purpose of dynamic analysis, it is often advantageous to determine displacement in terms of free vibration modes:

$$v(x, t) = \sum_{n=1}^{\infty} y_n(t)\phi_n(x), \quad 0 \leq x \leq L, \quad 0 \leq t \leq \infty$$

(3)

where $\phi_n(x) = \text{modes that constitute independent displacement patterns}$, and $y_n(t) = \text{amplitudes of the modes that serve as generalized coordinates}$. The modes serve the same purpose as the trigonometric functions in a Fourier series, and they are used
for the similar reasons that (i) they possess orthogonality properties and (ii) they are efficient in the sense that the exact displacements at all time could be approximated with sufficient accuracy by employing only a few modes.

As shown in textbooks (e.g. Clough and Penzien [10]) the mode shapes for a simply supported beam can be derived as:

$$\phi_n(x) = \sin \frac{n\pi x}{L}, \quad n = 1, \cdots, \infty$$  \hspace{1cm} (4)

and the corresponding frequencies as:

$$\omega_n = n^2 \pi^2 \sqrt{\frac{EI}{mL^4}}, \quad n = 1, \cdots, \infty$$  \hspace{1cm} (5)

For the $n$th mode, the corresponding generalized mass is obtained as:

$$M_n = \int_0^L m \phi_n(x)^2 \, dx = \frac{mL}{2}$$  \hspace{1cm} (6)

and the corresponding generalized force is:

$$P_n(t) = \int_0^L q(x, t) \phi_n(x) \, dx, \quad 0 \leq t \leq \frac{L}{c}$$  \hspace{1cm} (7)

Substituting Eq. (2) into Eq. (7) yields

$$P_n(t) = \int_0^L W \delta(x - ct) \phi_n(x) \, dx = W \phi_n(ct), \quad 0 \leq t \leq \frac{L}{c}$$  \hspace{1cm} (8)

From Eq. (4) and Eq. (8), one obtains:

$$P_n(t) = W \sin \Omega t, \quad 0 \leq t \leq \frac{L}{c}$$  \hspace{1cm} (9)

where $\Omega = \frac{n \pi c}{L}$. One can solve for $y_n(t)$ from the equation of motion corresponding to a SDOF system as:

$$\ddot{y}_n(t) + 2\xi_n \omega_n \dot{y}_n(t) + \omega_n^2 y_n(t) = \frac{2W}{mL} \sin \Omega t, \quad 0 \leq t \leq \frac{L}{c}$$  \hspace{1cm} (10)

where $\xi_n$ = the damping ratio that is included to account for the physical reality of energy dissipation. The analytical solution of Eq. (10) can be obtained easily. However, for simplicity in presentation, considering $\xi_n = 0$, if the system starts from "at rest" initial conditions, one has:

$$y_n(t) = \frac{2W}{mL} \frac{1}{\omega_n^2 - \Omega_n^2} \left( \sin \Omega_n t - \frac{\Omega_n}{\omega_n} \sin \omega_n t \right), \quad 0 \leq t \leq \frac{L}{c}$$  \hspace{1cm} (11)
From Eqs. (3), (4) and (11), the deflection of a simply supported beam without damping due to a constant force moving across the beam with a constant velocity, $c$, can be described as [7]:

$$v(x, t) = \frac{2W}{mL} \sum_{n=1}^{\infty} \frac{1}{\omega_n^2 - \Omega_n^2} \left( \sin \Omega_n t - \frac{\Omega_n}{\omega_n} \sin \omega_n t \right) \sin \frac{n\pi x}{L}, \quad 0 \leq t \leq \frac{L}{c}$$ (12)

The above equation applies only while the constant force is still on the bridge span, i.e. $t \leq \frac{L}{c}$. For the era, $t > \frac{L}{c}$, the response is determined simply by computing the state conditions at the instant that the force leaves the span and using these as initial conditions for a free vibration analysis.

**Numerical Solution Procedure**

Eq. (12) is only valid for a beam with uniform characteristics. When considering a damaged beam, modeled based on beam rigidity reduction at a segment of the beam, it is necessary to employ a numerical solution procedure. The finite element method (FEM) serves as a suitable approach.

The application of FEM converts the partial differential equation, Eq. (1), into a finite number of coupled second order ordinary differential equations:

$$[M]\{\ddot{v}\} + [K]\{v\} = \{P(t)\}$$ (13)

where $[M]$ and $[K]$ represent the global mass and stiffness matrices, $\{\ddot{v}\}$ and $\{v\}$ are space-discretized time-dependent acceleration and displacement arrays, and $\{P(t)\}$ is the corresponding forcing array. In this study, damage is modeled based on multiplying a stiffness reduction factor to local stiffness matrices corresponding to a specified location. It amounts to a reduction on the rigidity of the beam at a specified location. To compute the solution, it is more efficient to uncouple the equations associated with the MDOF system given in Eq. (13), and solve each resulting equation sequentially as a single degree of freedom (SDOF) system. This is accomplished via the mode superposition technique, a method that has been described previously in the analytical solution procedure for a continuous system. The nodal displacement, $\{v\}$, can be expressed in terms of mode shapes, $\{\hat{\phi}_n\}$, and the corresponding modal displacements, $y_n$:

$$\{v\} = \sum_{n=1}^{N} y_n \{\hat{\phi}_n\}$$ (14)

where $N$ is the number of modes being considered. The mode shapes $\{\hat{\phi}_n\}$ (normalized by the mass matrix, i.e. $\{\hat{\phi}_n\}^T [M] \{\hat{\phi}_n\} = 1$, in which the superscript $T$ is the
transpose operator) and the corresponding frequency \( \omega_n \) are obtained via an eigen analysis associated with \([M]\) and \([K]\), and \( y_n \) is solved from the SDOF system:

\[
\ddot{y}_n + 2\xi_n\omega_n\dot{y}_n + \omega_n^2 y_n = P_n^*(t) \quad 0 \leq t \leq \frac{L}{c}
\]

in which \( \xi_n \) = damping ratio (added to account for the energy loss via vibration), and the \( n \)th generalized force \( P_n^*(t) \) should be obtained from the \( n \)th mode based on the principle of Eq. (8). To numerically solve the SDOF system, the piecewise linear interpolation method will be employed in this study.

**Matched Damage Processing**

A newly proposed damage localization and assessment method, named matched damage processing (MDP), is presented below. The concept of MDP builds upon that of the matched field processing (MFP) used by the underwater acoustic community. The underlying principle of MDP is simple and can be illustrated by the following example. Considering a simply supported beam with an applied load and an unknown damage location and damage level, one can perform MDP as follows:

1. Collect response measurements from one or many sensors;
2. Compute the corresponding response based on a damaged beam model that selects damage locations and damage levels systematically;
3. Compare or correlate the measured response with the modeled response.
4. Select the location and level showing the highest agreement or correlation to be the estimated location and level of damage.

The key component of an MDP technique is the comparison algorithm. Several factors must be considered when choosing an appropriate comparison method. Comparison algorithms may use either the time or frequency domain. Signals are generally collected as time series; therefore a time domain comparison might be straightforward. However, this time domain comparison is sensitive to the time shift between the measured and modeled signals. Time shifts are usually less influential if a frequency domain comparison is conducted. The number of sensors available is also an important factor to decide comparison algorithms. Sophisticated comparison algorithms might be developed if multiple sensors are available. Usually, using a multiple sensor algorithm tends to achieve greater capability and accuracy.
In this study, a simple least squares (LS) processor is applied. This processor squares the difference between the measured and modeled response time series. The LS processor is to estimate the damage location and damage degree based on a minimal value of $\epsilon^2$ mathematically represented as:

$$
\epsilon^2 = \sum_{i=1}^{N} \sum_{j=1}^{M} [\hat{v}(x_i, t_j) - v(x_i, t_j)]^2
$$

where $N =$ number of sensors, $M =$ number of time steps for each signal, $x_i =$ the location of the $i$th sensor, and $\hat{v}$ and $v =$ the measured and modeled response, respectively.

**Numerical Examples**

Three examples that apply the LS processor to different simulated measurements are presented below. All examples consider one hundred scenarios, comprised of ten damage locations (the beam is divided spatially into ten equal segments as shown in Figure 1) and ten levels of damage (stiffness reduction factors are from 10% to 90% with increment of 10%). Example I considers a case where the simulated measurement precisely matches one of the 100 modeled scenarios. Example II considers that the simulated measurement does not match any of the modeled scenarios. In Example III, white noise is added to the simulated measurements. Only displacement collected from a single sensor at mid-span is used for the numerical examples.

Figure 1: Simply Supported Beam with Damage Applied at Location 3
Example I

The first example considers a case where the damage occurs at location 3 as shown in Figure 1. It is of interest to gain some insight as to the variations of modal frequencies and mode shapes between undamaged and damaged beams before proceeding with the proposed MDP technique. Table 1 is a comparison of the structure’s modal frequencies for the baseline (undamaged) model, denoted as $\omega_B$, and those of 10%, 50%, and 90% damage level, denoted as $\omega_{10\%}$, $\omega_{50\%}$ and $\omega_{90\%}$, respectively, for modes 1-5, 10, 20, and 30. The normalized modal frequencies (denoted as $\bar{\omega}_B$) of the undamaged beam, listed in column 2, are normalized by its first modal frequency. Those normalized frequencies are obtained directly from a FEM application, and they match well with the corresponding analytical solutions (they should be $n^2$, where $n$ is the mode number), except for minor discrepancies for modes 20 and 30. The small discrepancies are due to the fact that only 120 elements were used in the FEM mesh. The remaining columns display the ratio of modal frequencies for the damaged to those undamaged structures. The results suggest that one can observe noticeable change in modal frequencies only for cases with high degree of damage. Even with a noticeable change in modal frequencies, it is realized that the frequency shifts can offer only the possible existence of damage. In particular, no information about the location and severity of the damage can be determined from the change of frequencies. Figures 2 and 3 examine the mode shapes for the undamaged and 70% damaged beam for modes 1-5, 10, 20, and 30. In general, they cannot offer distinct information about the severity or location of the damage either. It is clear that a more robust damage detection algorithm is desired.

Figure 4 is the result of MDP applied to a measured beam with a 50% reduction in stiffness at Location 3. In contrast to the column 4 of Table 1 where very little shift in the modal frequencies is revealed, the MDP contour plot allows one to visually identify the location and severity of the damage. The calculated scenario with 50% damage at Location 3 has a significantly lower $\epsilon^2$ value than any of the other models. Therefore, one can easily conclude from this result that the measured beam has a 50% reduction of stiffness at Location 3. For a lower level of damage, with 10% in stiffness reduction, as shown in the column 3 of Table 1 which indicates that no noticeable shift in frequencies exist. This insignificant frequency change is likely to mislead one to conclude that this beam structure has sustained no damage. However, the MDP plot as shown in Figure 5 again provides a clear visualization of the location and severity of the damage. For a high level of damage case with 90% in stiffness reduction, although significant frequency shifts do occur, an estimation of location and severity from this
Table 1: Comparison of Mode Frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\bar{\omega}_B$</th>
<th>$\omega_{10%}^B$</th>
<th>$\omega_{50%}^B$</th>
<th>$\omega_{90%}^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.00</td>
<td>0.95</td>
<td>0.71</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.99</td>
<td>0.92</td>
<td>0.72</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>0.99</td>
<td>0.96</td>
<td>0.88</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>1.00</td>
<td>0.99</td>
<td>0.90</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>0.99</td>
<td>0.96</td>
<td>0.86</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>0.99</td>
<td>0.97</td>
<td>0.87</td>
</tr>
<tr>
<td>20</td>
<td>403</td>
<td>0.99</td>
<td>0.97</td>
<td>0.87</td>
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<tr>
<td>30</td>
<td>906</td>
<td>0.99</td>
<td>0.97</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Figure 2: Shapes of Modes 1-4
Figure 3: Shapes of Modes 5, 10, 20, 30
Figure 4: MDP Plot: 50% Damage at Location 3
information alone is still not possible. The observation of the corresponding MDP plot shown in Figure 6 allows one to conclude with confidence that 90% damage has occurred at Location 3 of the measured structure. Clearly, under ideal situations as assumed in the present example, applying MDP allows one to gain greater insight into the status of the beam structure than monitoring the structure’s modal parameters.

Example II

As expected, MDP must be effective when the measured structure precisely matches one of the calculated scenarios; this perfect match between a physical structure and its mathematical model is unlikely to occur in reality. It is therefore of interest to examine the effectiveness of MDP in a less than ideal condition. Example II considers a situation where the damage level or location of the measured beam does not precisely match the modeled scenarios. In the following numerical exercise for Example II, damage is applied to an area of the measured beam within Location 5 as indicated
Figure 6: MDP Plot: 90% Damage at Location 3

Figure 7: Simply Supported Beam with Damage Applied within Location 5
in Figure 7. The corresponding MDP plots for damage level equal to 50%, 10% and 90% are presented in Figures 8, 9, and 10 respectively.

![Figure 8: MDP Plot: 50% Damage within Location 5](image)

Figure 8: MDP Plot: 50% Damage within Location 5

From Figure 8, one observes that the lowest $\epsilon^2$ value occurs at the calculated scenario with 30% damage at Location 5. At first glance, this result may prompt one to reach a less positive conclusion about the accuracy of the MDP method, because the simulated damage is based on 50% damage within Location 5. However, if one notices that only a portion of Location 5 is affected by the stiffness reduction, it is logical to conclude that the equivalent damage level for the whole Location 5 must be less than 50%. Furthermore, it is noteworthy that the damage location is still correctly indicated by Figure 8.

Figure 9 is the resulting MDP plot for a 10% damage level case. A low $\epsilon^2$ value occurs at Locations 5 and 6 with damage level less than 10%. Judged from general standards, it is reasonable to say that the MDP has successfully estimated the approximate location and level of damage. For extremely high damage case (90% damage
level), the resulting MDP plot shown in Figure 10, it is found that MDP estimates a lower than 90% damage level at Location 5. The results are considered reasonable by applying the same logic as discussed for the 50% damage level case. Due to the fact that none of the model considered in Example II matches perfectly with the measured structure, it is of no surprise that one cannot perfectly estimate the damage location and severity from the MDP method. However, the MDP method does give a very good indication about the precise damage situation. In fact, one can rely on the above MDP plots to “zoom in” the MDP plot for the next round by eliminating unlikely scenarios and refining resolution to likely candidates. Doing this would allow one to more precisely approximate damage characteristics. The efficiency of MDP thus can be enhanced.
Figure 10: MDP Plot: 90% Damage within Location 5
Example III

Example III studies the effectiveness of MDP for situations that white noise random errors are injected to the measurements. The target damage location considered in this example is the same as that in Example I. The standard deviation associated with the random measurement noise is considered equal to 10% of the maximum measurement signal. The corresponding MDP plots for damage level equal to 50%, 90% and 10% are presented in Figures 11, 12, and 13 respectively. In general, for the 50% and 90% damage cases, applying MDP method can correctly identify the damage location and severity. For the 10% damage case, there is no clear indication exhibited from the MDP contour plot. Failing to identify the damage location for 10% case is due to the fact that the measurement noise level is too large in comparison with the damage level. Virtually, the 10% damage is buried in the noise and thus becomes unidentified. One should feel positive about the MDP method even under this case, because the resulting contour plot indeed suggests that the damage is small. It basically rules out
Figure 12: MDP Plot: 90% Damage at Locations 3 with noise
Figure 13: MDP Plot: 10% Damage at Locations 3 with noise

the likelihood for any large damage existing in the structure. One should also realize that measurement errors usually are uncorrelated random errors and that the present example takes measurements only from one location for a short duration. If one utilizes a significant number of sensors at various locations to collect measurements for longer period of time while applying the MDP method, it is theoretically possible that all random errors can cancel each other, therefore the 10% damage (can be viewed as a biased term) is likely observable.

Concluding Remarks

The framework of a new structural health monitoring method — Matched Damage Processing (MDP) — has been developed in this study. The concepts of MDP builds upon the knowledge of Matched Field Processing already developed in the underwater acoustics community. The output of MDP is an image or a contour plot representing
the agreement between the physical measurements (in the study simulated measurements are used) and their counterparts computed based on specific models that have been systematically chosen to possess different damage location and damage level. Because the MDP method offers a visual solution, instead of “calculating” a target damage location or damage level directly, it apparently is more robust than other existing approaches, such as methods based on calculating modal parameters.

Three classes of numerical examples are demonstrated in this article. Overall, the MDP method appears to be a promising method for global health monitoring to determine the damage location and severity. Since the MDP method is introduced for the first time in this paper, the authors believe that a number of related issues could be furthered improved or developed. For instance, a better comparison processor can be developed to replace the simple least-square processor employed in the present study. Using advanced data analysis methods and applying array signal processing techniques (that need multiple sensors) could increase the effectiveness or reliability of the MDP method. Ultimately, it is desired to apply the method to realistic bridge structures where the complexity and uncertainty associated with numerical models must be properly addressed.
References


